Network character and filter convergence in $\beta\omega$

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Filters on ω

Definition

A family $\mathcal{F}$ of non-empty subsets of ω is a *filter* if $\mathcal{F}$ is closed under intersections and taking supersets.

A filter $\mathcal{F}$ is *free* if $\cap\mathcal{F} = \emptyset$.

For a filter $\mathcal{F}$ on ω let $\mathcal{F}^+ = \{A \subset \omega : \forall F \in \mathcal{F}\ \ A \cap F \neq \emptyset\}$.

Basic Example

The Fréchet filter $\mathfrak{Fr} = \{\omega \setminus F : F \text{ is finite}\}$ of cofinite subsets.

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Let $\mathcal{F}$ be a free filter on ω.

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A sequence $(x_n)_{n\in\omega}$ in a topological space X *$\mathcal{F}$-converges* to a point $x_\infty \in X$ if $\forall O(x_\infty)\ \exists F \in \mathcal{F}\ \forall n \in F\ \ x_n \in O(x_\infty)$.

Remark

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Two Motivating Facts:

1. Any sequence in a compact Hausdorff space is $\mathcal{U}$-convergent for any ultrafilter $\mathcal{U}$.
2. No injective sequence in $\beta\omega$ is $\mathfrak{Fr}$-convergent for the Fréchet filter $\mathfrak{Fr}$.

Problem

Which property of the Fréchet filter $\mathfrak{Fr}$ is responsible for such a phenomenon?

The same in other words:

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What can be said about filters $\mathcal{F}$ admitting an injective $\mathcal{F}$-convergent sequence in $\beta\omega$?

We shall check three types of properties of filters: topological, measure-theoretic and combinatorial.

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Topological properties of filters

Each filter $\mathcal{F}$ on ω is a subset of the power-set $\mathcal{P}(\omega) = 2^\omega$, which carries a nice compact metrizable topology.

So, we can speak about topological properties of filters considered as subsets of the Cantor cube $\mathcal{P}(\omega) = 2^\omega$.

Definition

A filter $\mathcal{F}$ on ω is called *meager* (*analytic*, F_σ) if $\mathcal{F}$ is meager (analytic, F_σ) subset of 2^ω.

It is clear that

$$\mathfrak{Fr} \;\Rightarrow\; F_\sigma \;\Rightarrow\; \text{analytic} \;\Rightarrow\; \text{meager}$$

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The Cantor set 2^ω also carries a nice Borel probability measure called the Haar measure, so we can speak about measure-theoretic properties of filters.

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A filter $\mathcal{F}$ on ω is

- *measurable* if it is measurable with respect to the Haar measure;
- *null* if it has Haar measure null.

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Combinatorial properties of filters

A set A is called a *pseudointersection* of a family of sets $\mathcal{F}$ if for every $F \in \mathcal{F}$ $A \subset^* F$ (which means that $A \setminus F$ is finite).

Definition

A filter $\mathcal{F}$ on ω is called a P-filter (a P^+-filter) if each countable subfamily $\mathcal{C} \subset \mathcal{F}$ has a pseudointersection A in $\mathcal{F}$ (in $\mathcal{F}^+$).

The *character* $\chi(\mathcal{F})$ of a filter $\mathcal{F}$ is the smallest cardinality of a base of $\mathcal{F}$.
The smallest character of a free ultrafilter is denoted by $\mathfrak{u}$.

Theorem (Ketonen)

Any filter $\mathcal{F}$ of character $\chi(\mathcal{F}) < \mathfrak{d}$ is a P^+-filter.

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On character of points in separable subspaces of $\beta\omega$

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If a free filter $\mathcal{F}$ admits an injective $\mathcal{F}$-convergent sequence in $\beta\omega$, then $\chi(\mathcal{F}) \geq \min\{\mathfrak{u}, \mathfrak{d}\}$.

Corollary

For any separable subspace $X \subset \beta\omega$ and any non-isolated point $x \in X$ we have $\chi(x, X) \geq \min\{\mathfrak{d}, \mathfrak{u}\}$.

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Let X be a (separable) subspace of $\beta\omega$ and $x \in X$. Is $\chi(x, X) \geq \mathfrak{u}$?

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Proof of two Theorems

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If a free filter $\mathcal{F}$ admits an injective $\mathcal{F}$-convergent sequence (x_n) in $\beta\omega$, then $\chi(\mathcal{F}) \geq \min\{\mathfrak{u}, \mathfrak{d}\}$.

Proof.

If $\chi(\mathcal{F}) < \min\{\mathfrak{d}, \mathfrak{u}\}$, then $\chi(\mathcal{F}) < \mathfrak{d}$ and by Ketonen's Theorem, $\mathcal{F}$ is a P^+-filter.
This implies the existence of a subset $A \in \mathcal{F}^+$ such that the subsequence $\{x_n\}_{n \in A}$ is discrete. This subsequence is $\mathcal{F}|A$-convergent for the filter $\mathcal{F}|A = \{F \cap A : F \in \mathcal{F}\}$ which is an ultrafilter as $\{x_n\}_{n \in A}$ is discrete. Then $\mathfrak{u} \leq \chi(\mathcal{F}|A) \leq \chi(\mathcal{F})$. □

If $\mathcal{F}$ is an analytic P^+-filter, then $\mathcal{F}|A$ is analytic and cannot be an ultrafilter.
This gives a proof of the following theorem of J. Verner.

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$$\mathfrak{F}r \Rightarrow F_\sigma \Rightarrow \text{analytic } P^+ \Rightarrow \text{analytic} \Rightarrow \text{meager \& null}$$

Theorem (J.Verner, 2011)

A free filter $\mathcal{F}$ admitting an injective $\mathcal{F}$-convergent sequence in $\beta\omega$

- *is not an analytic P^+-filter;*
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Problem

Does $\beta\omega$ contain an injective $\mathcal{F}$-convergent sequence for

- *an analytic filter $\mathcal{F}$?*
  (This is open but we believe that the answer is No!?);
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  (Here we have a surprising answer: Yes!)

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A surprising Theorem

Theorem

Each infinite compact Hausdorff space X contains an injective $\mathcal{F}$-convergent sequence for some filter $\mathcal{F}$, which is meager and null.

Talagrand's Characterization of meager filters

A function $\varphi : \omega \to \omega$ is *finite-to-one* if for every $y \in \omega$ the preimage $\varphi^{-1}(y)$ is finite and non-empty.

Theorem (Talagrand, 1980)

A filter $\mathcal{F}$ on ω is meager if and only if $\varphi(\mathcal{F}) = \mathfrak{Fr}$ for some finite-to-one function $\varphi : \omega \to \omega$. In this case we shall say that $\mathcal{F}$ is φ-meager.

Easy Observation: If $\varphi : \omega \to \omega$ is a finite-to-one function with

$$\sum_{n \in \omega} 2^{-|\varphi^{-1}(n)|} = \infty,$$

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Network character

Definition

A family $\mathcal{N}$ of subset of a topological space X is a *network at a point* $x \in X$ if each neighborhood $O(x)$ contains some set $N \in \mathcal{N}$.
If each set $N \in \mathcal{N}$ in open in X, then $\mathcal{N}$ is called a *π-base* at x.

Definition

The *network character* $nw_\chi(x, X)$ of X at a non-isolated point x is the smallest cardinality $|\mathcal{N}|$ of a network $\mathcal{N}$ at x that consists of infinite subsets of X.
For an isolated point $x \in X$ we put $nw_\chi(x, X) = 1$.

Easy Observations: 1) $\mathrm{nw}_\chi(x, X) \le \chi(x, X)$.
2) If X has no isolated points, then $\mathrm{nw}_\chi(x, X) \le \pi\chi(x, X)$.

Here the *π-character* $\pi\chi(x, X)$ is equal to the smallest cardinality of a π-base at x.

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A family $\mathcal{N}$ of subset of a topological space X is a *network at a point* $x \in X$ if each neighborhood $O(x)$ contains some set $N \in \mathcal{N}$. If each set $N \in \mathcal{N}$ in open in X, then $\mathcal{N}$ is called a *π-base* at x.

Definition

The *network character* $nw_\chi(x, X)$ of X at a non-isolated point x is the smallest cardinality $|\mathcal{N}|$ of a network $\mathcal{N}$ at x that consists of infinite subsets of X.
For an isolated point $x \in X$ we put $nw_\chi(x, X) = 1$.

Easy Observations: 1) $\mathrm{nw}_\chi(x, X) \leq \chi(x, X)$.
2) If X has no isolated points, then $\mathrm{nw}_\chi(x, X) \leq \pi\chi(x, X)$.

Here the *π-character* $\pi\chi(x, X)$ is equal to the smallest cardinality of a π-base at x.

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Network character of points in $\beta\omega$

It is well-known that $\omega^* = \beta\omega \setminus \omega$ has

- uncountable character $\chi(x, \beta\omega) \geq \mathfrak{u}$ and
- uncountable π-character $\pi\chi(x, \omega^*) \geq \mathfrak{r}$

at each point x.

Problem

Is network character $\mathrm{nw}_\chi(x, \beta\omega) \geq \aleph_1$ for all points $x \in \omega^*$?

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Surprising Answer: No!

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Is network character $\mathrm{nw}_\chi(x, \beta\omega) \geq \aleph_1$ for all points $x \in \omega^*$?

Surprising Answer: No!

Theorem

Each infinite compact Hausdorff space X contains a point $x \in X$ with $\mathrm{nw}_\chi(x, X) = \aleph_0$.

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If X is scattered, then X contains an injective convergent sequence (x_n) and hence $\mathrm{nw}_\chi(x, X) = \aleph_0$ for $x = \lim_{n\to\infty} x_n$.
If X is not scattered, then X admits a surjective continuous map $f : X \to \mathbb{I}$ onto $\mathbb{I} = [0, 1]$. Choose a closed subset $A \subset X$ such that $f|A$ is irreducible in the sense that $f(A) = \mathbb{I}$ but $f(B) \neq \mathbb{I}$ for any proper subset $B \subset A$.
The irreducibility of $f|A$ implies that A has no isolated points and for any $x \in A$ we get

$$\aleph_0 \le \mathrm{nw}_\chi(x, X) \le \mathrm{nw}_\chi(x, A) \le \pi\chi(x, A) \le \pi w(A) \le \pi w(\mathbb{I}) \le \aleph_0.$$

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Meager-convergence and countable network character

Theorem

If $\mathrm{nw}_\chi(x,X)=\aleph_0$ *for some point* x*, then some injective sequence in* X *is* $\mathcal{F}$*-convergent to* x *for some meager and null filter* $\mathcal{F}$*.*

Proof.

Let $\{N_k : k\in\omega\}$ be a countable network at x that consists of infinite subsets of X.
Fix any finite-to-one function $\varphi:\omega\to\omega$ such that $\lim_{n\to\infty}|\varphi^{-1}(n)|=\infty$ and $\sum_{n\in\omega}2^{-|\varphi^{-1}(n)|}=\infty$.
By induction choose an injective sequence $(x_n)_{n\in\omega}$ such that for every $n\in\omega$ the set $\{x_k\}_{k\in\varphi^{-1}(n)}$ intersects each set N_i with $i<|\varphi^{-1}(n)|$.
Then the sequence (x_n) $\mathcal{F}$-converges to x for the filter

$$\mathcal{F}=\big\{\{n\in\omega : x_n\in O(x)\} : O(x)\text{ is a neighborhood of } x \text{ in } X\big\}$$

which is φ-meager and hence is meager and null. □

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Unexpected properties of $\beta\omega$

Corollary

The space $\beta\omega$ contains:

1. *a point x with countable network character $\mathrm{nw}_\chi(x, \beta\omega)$;*
2. *an injective $\mathcal{F}$-convergent sequence for a meager and null filter $\mathcal{F}$.*

Problem

Study the properties of the set of points with countable network character in $\omega^* = \beta\omega \setminus \omega$.

This set is dense and hence not meager in ω^*.
It contains no weak P-point and hence has empty interior.
What else?

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For topological spaces X, Y let $C_p(X, Y) \subset Y^X$ be the space of continuous function from X to Y endowed with the topology of pointwise convergence.

Theorem

If for some meager filter $\mathcal{F}$ a topological space X contains an injective $\mathcal{F}$-convergent sequence, then for each Tychonov path-connected space Y with $|Y| > 1$ the function space $C_p(X, Y)$ is meager.

Application to function spaces $C_p(X, Y)$

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Reference:

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On meager function spaces, network character and meager convergence in topological spaces,
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Thank you!